

CTEQ - Introduction to Monte Carlo

Lecture 3

MPI, Hadronization, and Non-perturbative Effects

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Overview

- lecture 1 - introduction and Monte Carlo techniques
- lecture 2 - matrix elements and parton showers
- **lecture 3 - multi-parton interactions, hadronization, and non-perturbative effects**

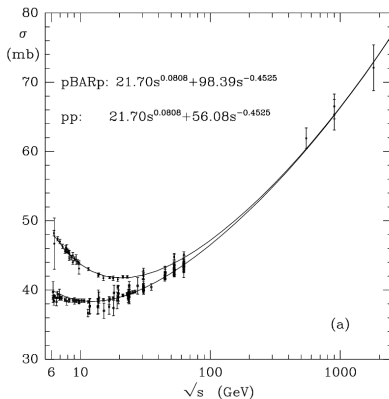
Soft Physics

- most measurements focus on hard processes, e.g. high mass dijet events
- soft QCD interactions occur in addition to hard processes
- events with only soft QCD are important for understanding QCD
- most events at the LHC are soft QCD \rightarrow minimum-bias events
- minimum-bias events require some detector activity
- no-bias events do not and randomly sample beam crossings
- from a theory perspective, minimum bias is sometimes taken as no bias

Total Cross Section

- use Donnachie-Landshoff parameterized Regge fit
- first term is Pomeron, second is Reggeon

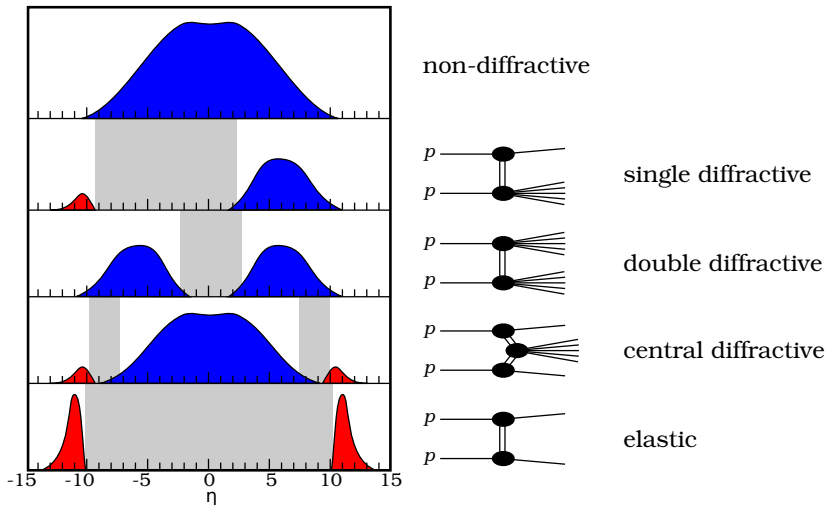
$$\sigma_{\text{tot}}(s) = (21.70s^{0.0808} + 56.08s^{-0.425}) \text{ mb}$$



- $\sigma_{\text{tot}}(14 \text{ TeV}) = 101.5 \text{ mb}$

Event Classification

- event classification different between experiment and theory



Non-diffractive Cross Section

$$\sigma_{\text{tot}}(s) = \sigma_{\text{inel}}(s) + \sigma_{\text{el}}(s)$$
$$\sigma_{\text{el}}(s) = \frac{\sigma_{\text{tot}}^2(s)}{16\pi(5 + 4s^{0.0808})}$$

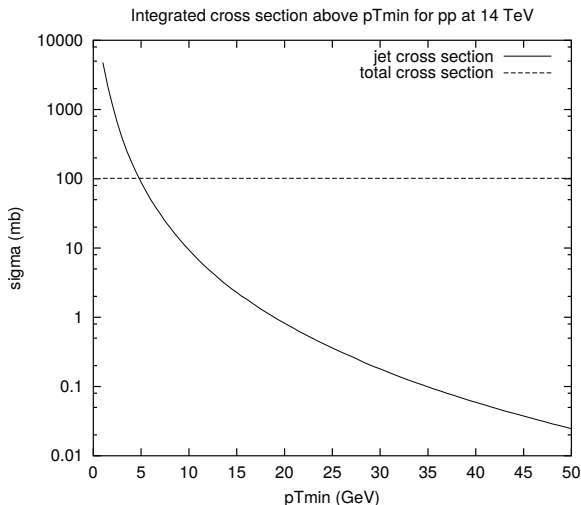
- relate σ_{tot} to σ_{el} via optical theorem and asymptotic behavior
- write non-diffractive cross section in terms of known inelastic and diffractive cross sections

$$\sigma_{\text{inel}}(s) = \sigma_{\text{ND}}(s) + 2\sigma_{\text{SD}}(s) + \sigma_{\text{DD}}(s) + \sigma_{\text{CD}}(s)$$
$$\Rightarrow \sigma_{\text{ND}}(s) = \sigma_{\text{inel}}(s) - 2\sigma_{\text{SD}}(s) + \sigma_{\text{DD}}(s) + \sigma_{\text{CD}}(s)$$

- minimum bias is primarily non-diffractive (with some double diffractive)

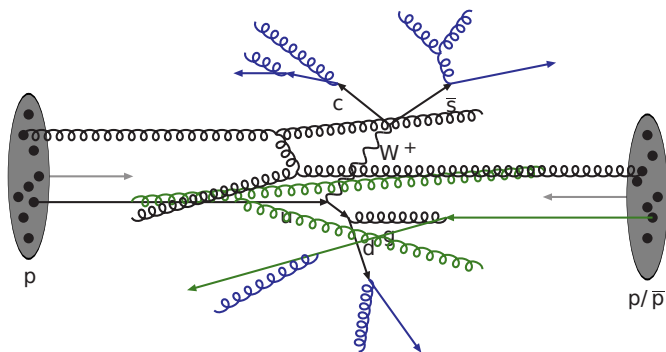
Modeling Minimum Bias

- from last lecture, QCD t -channel gluon-exchange cross sections diverge as $1/p_T^4$ for $\hat{t} \rightarrow 0$
- these processes will dominate minimum bias production



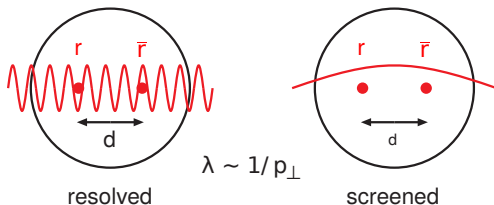
Modeling Minimum Bias

- QCD $2 \rightarrow 2$ jet cross section exceeds σ_{tot}
- interpret as multiple parton interactions (MPI) per event
- recast problem: number of MPI diverges, not cross section



Color Screening

- low p_T perturbative QCD not valid, q/g not asymptotic states
- assume some type of color screening in proton



- introduce smooth damping factor

$$\left(\frac{p_T^4}{p_{T0}^2 + p_T^2} \right) \left(\frac{\alpha_s(p_{T0}^2 + p_T^2)^2}{\alpha_s(p_T^2)} \right)$$

- allow p_{T0} to vary as a function of energy

$$p_{T0}(\sqrt{s}) = p_{T0}(E_0) \left(\frac{\sqrt{s}}{E_0} \right)^{\theta}$$

Multi-parton Interactions

- regulated $\sigma_{2 \rightarrow 2}$ is finite
- assuming independent interactions, number of MPI can be estimated

$$\begin{aligned}\langle n_{\text{MPI}} \rangle \sigma_{\text{ND}} &= \sigma_{2 \rightarrow 2} \\ \Rightarrow \langle n_{\text{MPI}} \rangle &= \frac{\sigma_{2 \rightarrow 2}}{\sigma_{\text{ND}}}\end{aligned}$$

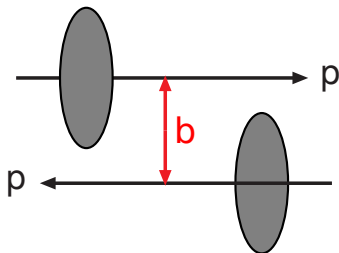
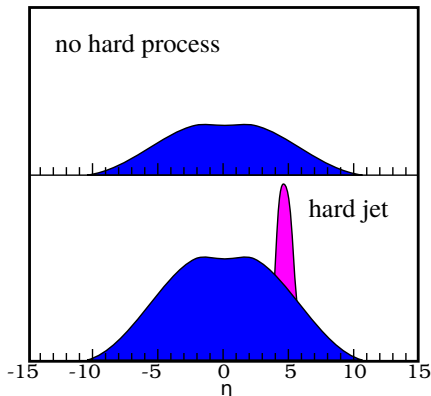
- can now write MPI probability

$$\frac{d\mathcal{P}_{\text{MPI}}}{dp_{\text{T}}} = \frac{1}{\sigma_{\text{ND}}} \frac{d\sigma_{2 \rightarrow 2}}{dp_{\text{T}}} \exp \left(- \int_{p_{\text{T}}}^{p_{\text{T}i-1}} \frac{1}{\sigma_{\text{ND}}} \frac{d\sigma_{2 \rightarrow 2}}{dp'_{\text{T}}} dp'_{\text{T}} \right)$$

- use veto algorithm to generate subsequent MPI

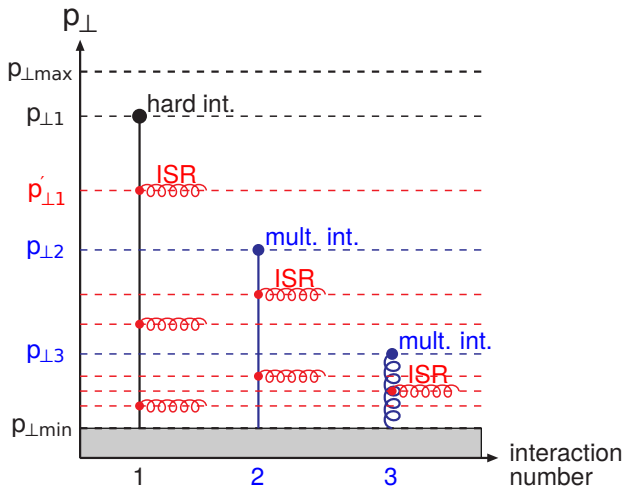
Some Complications

- MPI are not actually independent
- modify PDFs after MPI selected, include flavor conservation
- consider proton impact parameter
- hard process \rightarrow more underlying event (pedestal effect)



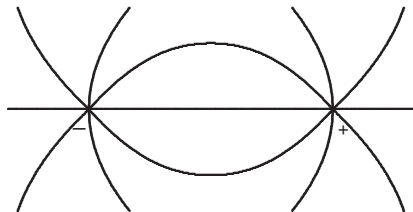
Putting it Together

- with p_T ordered parton shower, interleave ISR, FSR, and MPI

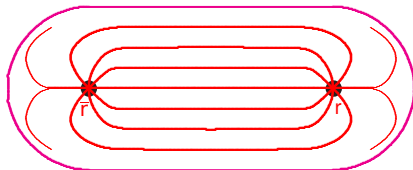


Field Lines

- QED field lines extend to infinity

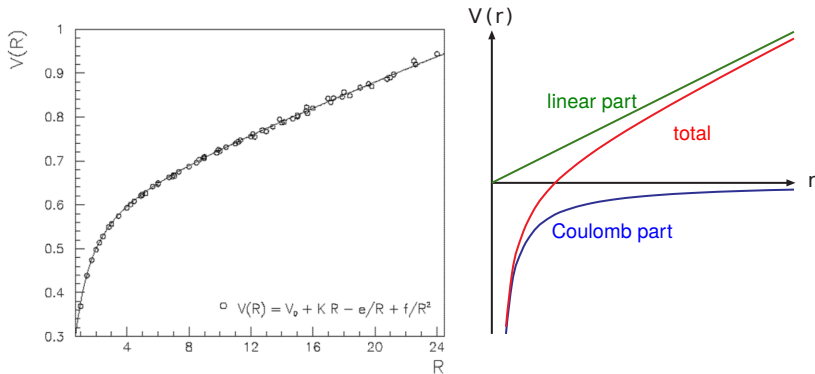


- QCD field lines compressed into strings



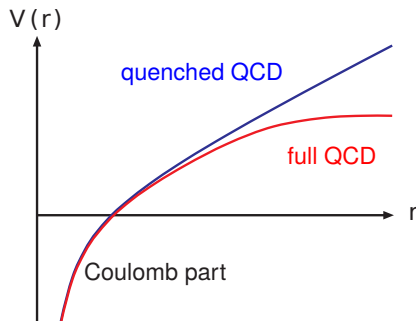
QCD Potential

- quenched lattice QCD demonstrates linear confinement



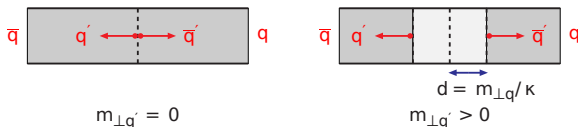
QCD Potential

- reality might not be quite so nice ...
- a non-quenched lattice measurement would be nice!



String Breaks

- each break is described by quantum tunneling
- only consider linear term



$$\mathcal{P} \propto \exp \left(\frac{-\pi m_{Tq}^2}{\kappa} \right)$$

$$m_T^2 = p_T^2 + m^2$$

$$\rightarrow \mathcal{P} \propto \exp \left(\frac{-\pi p_T^2}{\kappa} \right) \exp \left(\frac{-m_q^2}{\kappa} \right)$$

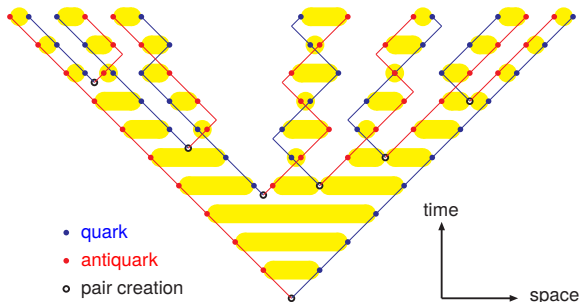
- common Gaussian p_T distribution for all breaks
- heavy quark production suppressed
 $u\bar{u} : d\bar{d} : s\bar{s} : c\bar{c} \approx 1 : 1 : 0.3 : 10^{-11}$
- baryons produced via diquark breaks

String Model

- 1 randomly select side of string for break
- 2 select flavor of produced hadron
- 3 perform string break, sample p_T
- 4 sample remaining momentum of the string, z

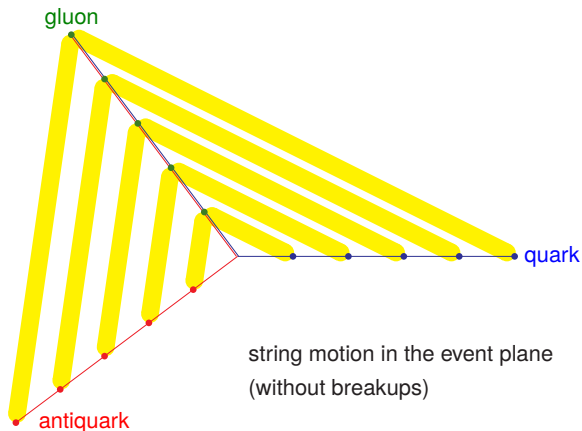
$$f(z) \propto \frac{1}{z} (1-z)^a \exp\left(-b \frac{m_T^2}{z}\right)$$

- 5 return to 1 until no remaining string momentum

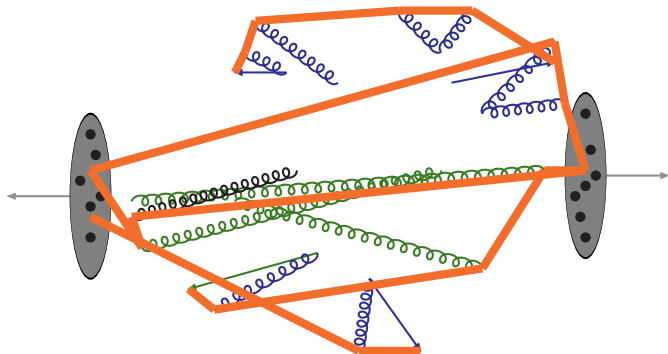


String Kinks

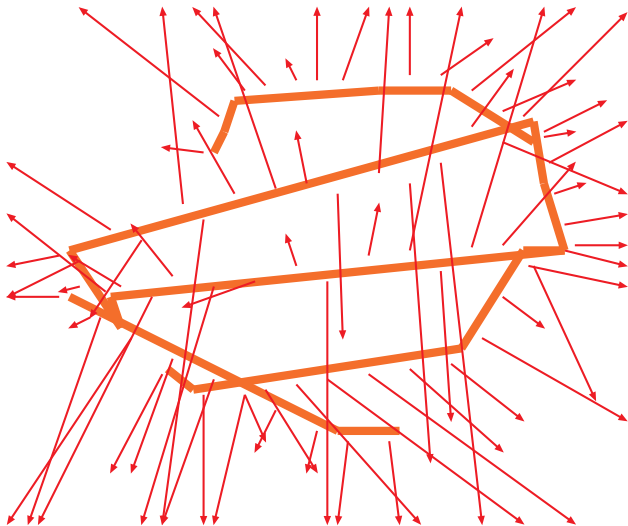
- more complex topologies exist (attached gluons, gluon-gluon, junctions, *etc.*)
- gluons on string provide a kink, modifying the kinematics



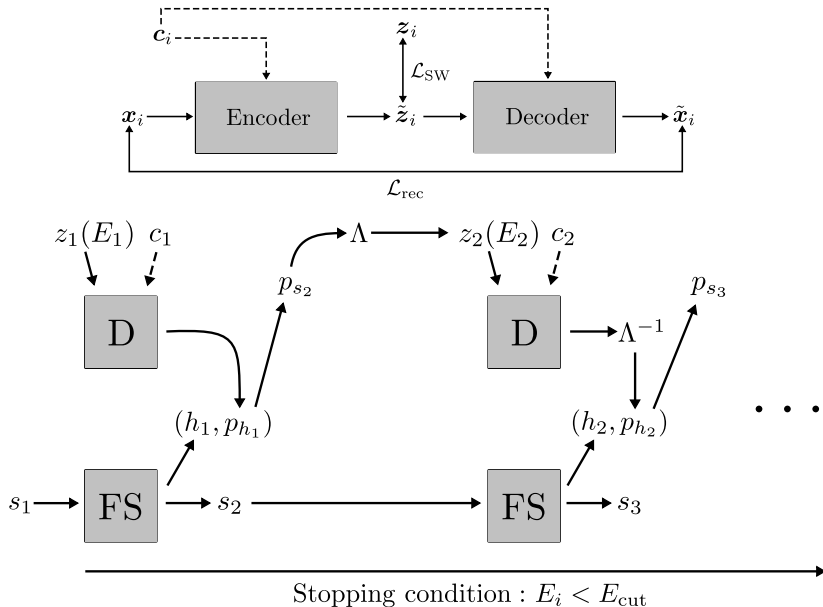
Strings in an Event



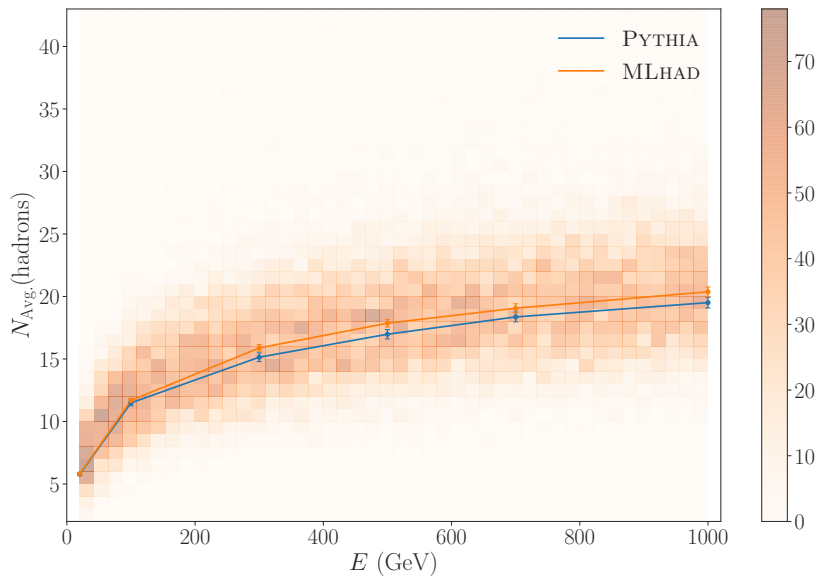
Hadrons in an Event



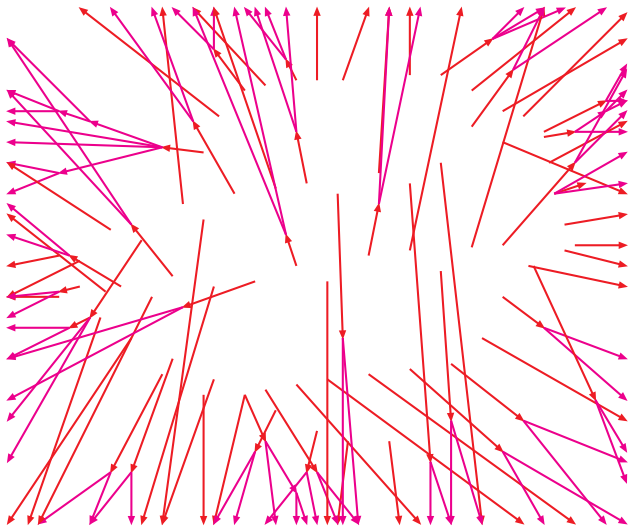
Machine Learning Hadronization



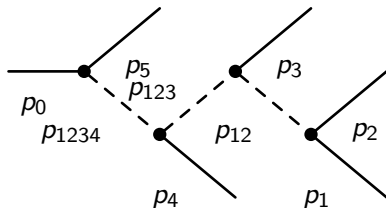
Machine Learning Hadronization



Decays of Hadrons



Phase-Space Generation



- m -generator
- two-body decays through intermediate masses

$$d\Phi_2(q_0, q_1, q_2) = \left(\frac{1}{(2\pi)^2 2^2} \right) \delta(q_0 - q_1 - q_2) \frac{d\vec{q}_1}{E_1} \frac{d\vec{q}_2}{E_2}$$

$$d\Phi_3(q_0, q_1, q_2, q_3) = \left(\frac{2}{\pi} \right) d\Phi_2(q_0, q_{12}, q_3) m_{12} dm_{12} d\Phi_2(q_{12}, q_1, q_2)$$

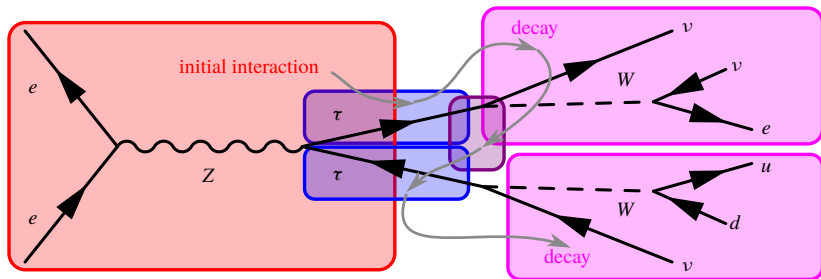
- weight by \mathcal{M} for the phase-space point

Spin Correlations

- $D \equiv$ decay matrix for each particle, $D_{\text{initial}} = \mathbb{I}$
- $\mathcal{M} \equiv$ matrix element, $\rho \equiv$ density matrix

- 1 calculate \mathcal{M} for the initial interaction
- 2 find ρ for an outgoing particle using the interaction \mathcal{M} and D 's of the remaining outgoing particles
- 3 decay the particle using its \mathcal{M} , ρ , and the D 's of its decay products
- 4 repeat 2 - 3 until all decay products are stable.
- 5 calculate D for the particle
- 6 go up a decay and perform 2 - 5 on the undecayed particles
- 7 Repeat 2 - 6 until all particles are decayed

Spin Correlations



$$\textcircled{2} \rho_{\lambda_j \lambda'_j}^j = \rho_{\kappa_1 \kappa'_1}^1 \rho_{\kappa_2 \kappa'_2}^2 \mathcal{M}_{\kappa_1 \kappa_2; \lambda_1 \dots \lambda_n} \mathcal{M}_{\kappa'_1 \kappa'_2; \lambda'_1 \dots \lambda'_n}^* \prod_{k \neq j} D_{\lambda_k \lambda'_k}^k$$

$$\textcircled{3} \mathcal{W}_{\text{decay}} = \rho_{\lambda_0 \lambda'_0} \mathcal{M}_{\lambda_0; \lambda_1 \dots \lambda_n} \mathcal{M}_{\lambda'_0; \lambda'_1 \dots \lambda'_n}^* \prod_{k=1, n} D_{\lambda_k \lambda'_k}^k$$

$$\textcircled{5} D_{\lambda_0 \lambda'_0} = \mathcal{M}_{\lambda_0; \lambda_1 \dots \lambda_n} \mathcal{M}_{\lambda'_0; \lambda'_1 \dots \lambda'_n}^* \prod_{l=1, n} D_{\lambda_l \lambda'_l}^l$$

Forming Deuterons

- how do deuterons form?
 - first guess, coalesce proton-neutron pairs into deuteron
- ① make all possible pn combinations
 - ② randomize ordering of combinations
 - ③ calculate k for each pn combination in COM frame

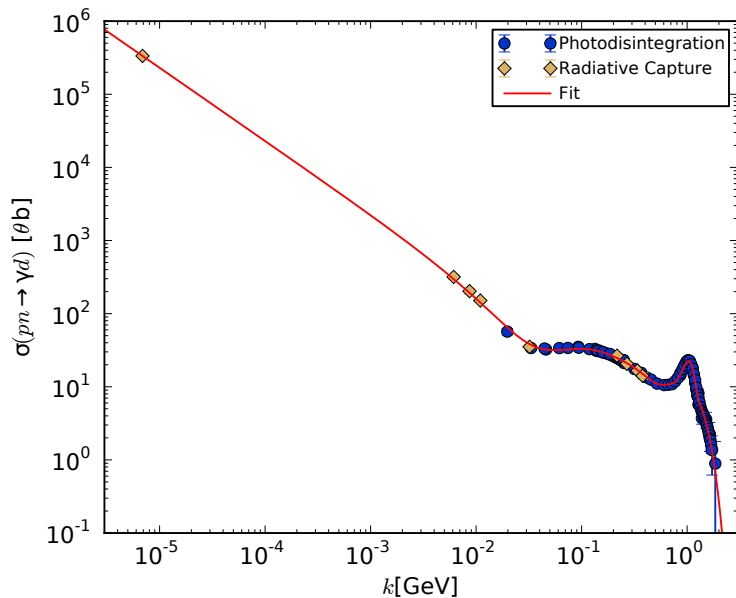
$$k = \sqrt{(\vec{p}_p - \vec{p}_n)^2}$$

- ④ if $k < c_0$ combine pn combination into deuteron

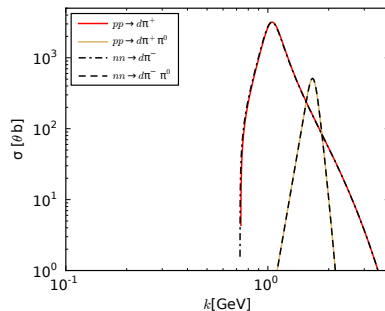
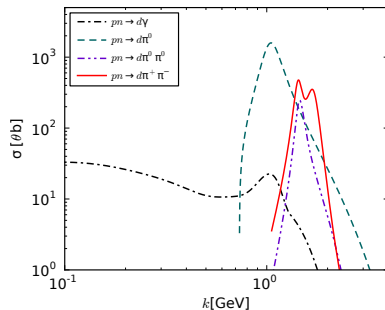
A More Sophisticated Model

- does not describe LHC data particularly well
- assumes uniform cross section in k
- does not handle energy-momentum conservation
- neglects other production channels
- next try, include all $2 \rightarrow n$ production, differential in k (Dal-Raklev model)
 - $pn \rightarrow \gamma D$
 - $pn \rightarrow \pi^0 D$
 - $pn \rightarrow \pi^- \pi^+ D$
 - $pn \rightarrow \pi^0 \pi^0 D$
 - $pp \rightarrow \pi^+ D$
 - $nn \rightarrow \pi^- D$
 - $pp \rightarrow \pi^+ \pi^0 D$
 - $nn \rightarrow \pi^- \pi^0 D$

Cross Sections from Data

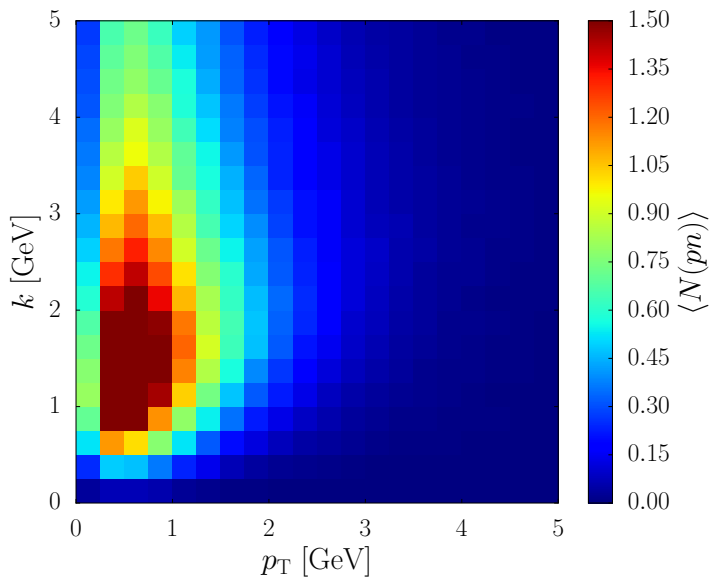


Cross Section Fits

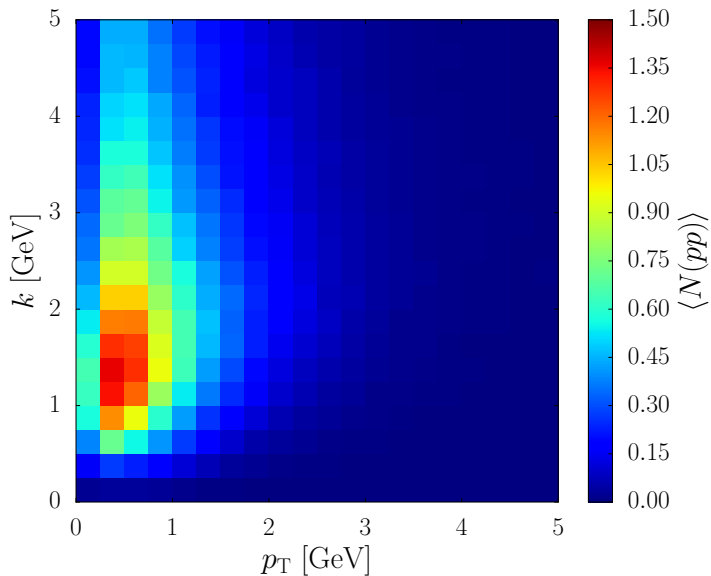


- all the cross sections are parameterized with relatively simple functions

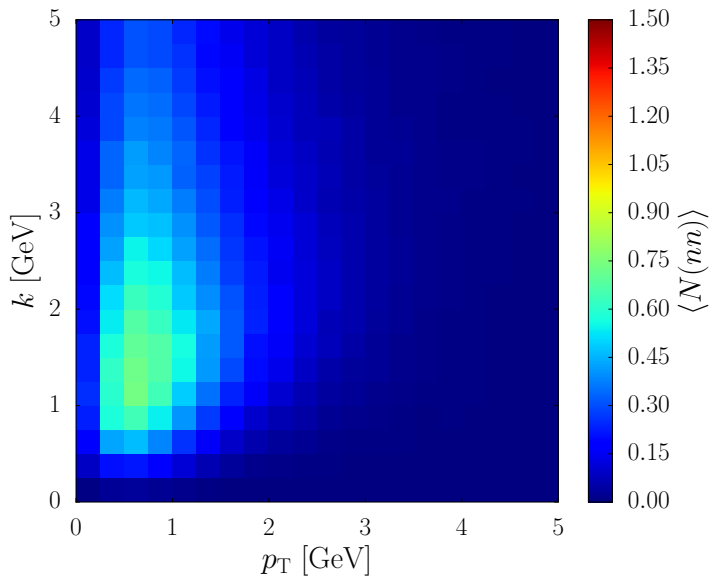
Possible Combinations



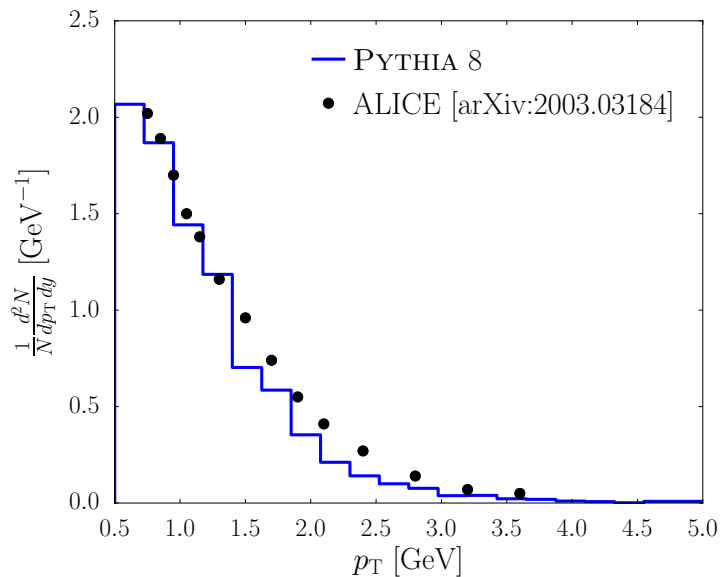
Possible Combinations



Possible Combinations

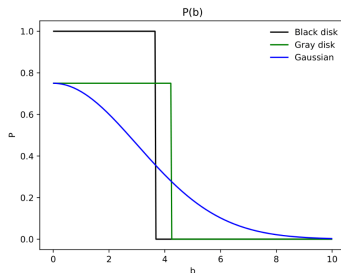
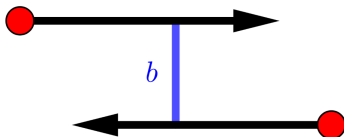


Does it Work?



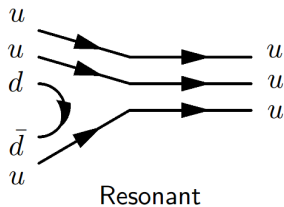
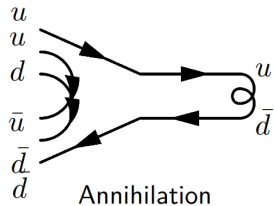
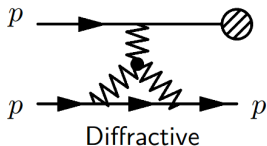
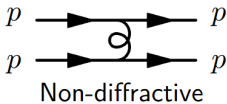
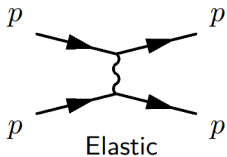
Spacetime Picture

- coalescence model only in momentum space
- if we have full spacetime picture of event, then can consider this instead
- hadronic rescattering occurs when two hadrons pass closely in spacetime



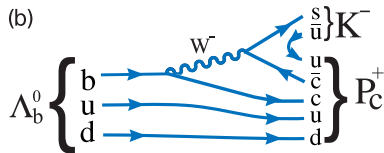
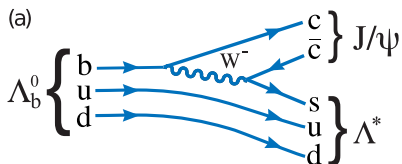
Hadronic Rescattering

- just like soft partonic QCD, we have similar interactions



Exotic Hadrons

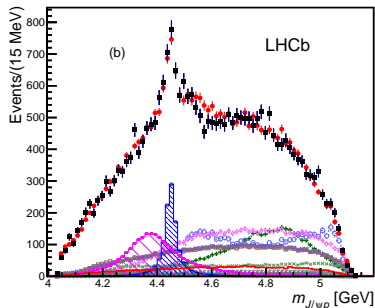
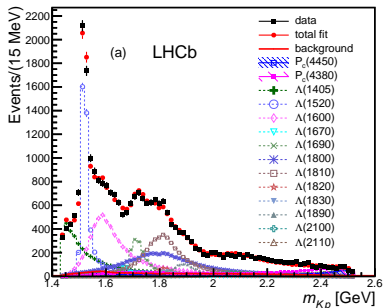
- LHCb set out to measure $\Lambda_b^0 \rightarrow J/\psi[\rightarrow \mu\mu]\Lambda^*[K^- p^+]$
- saw some unexpected structure



- nature of decay indicates five-quark states
- $P_c(4312)^+, P_c(4440)^+, P_c(4457)^+$

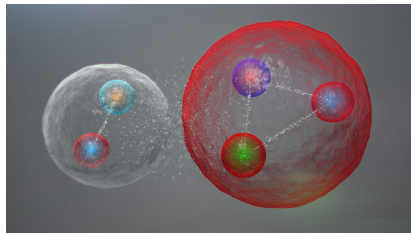
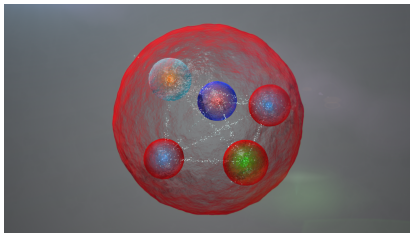
Show Me the Data!

- can look at different final state combinations
 $J/\psi K^- p^+$
- expect Λ^* resonance in $K^- p^+$
- do not expect resonance in $J/\psi p^+$



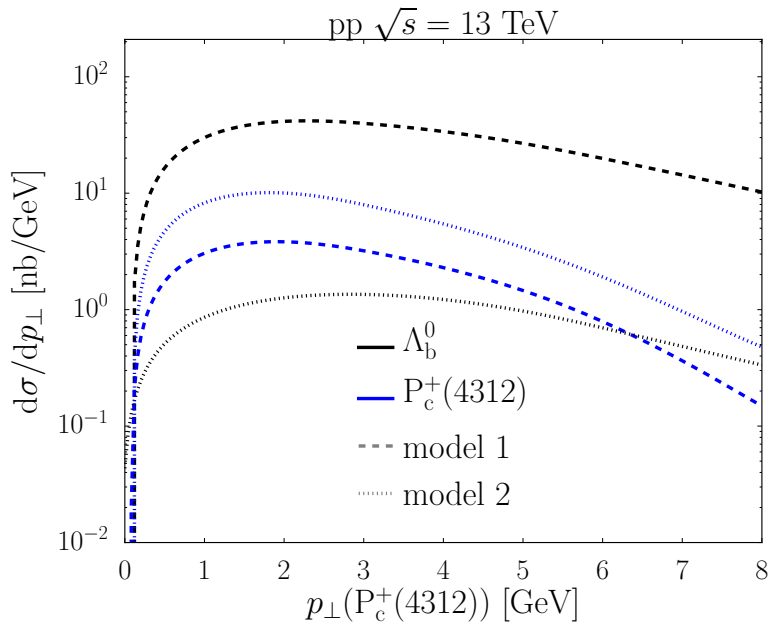
Exotic Models

- how do we model the pentaquarks?



- can consider either tightly bound, or more like molecular state
- possible $\Sigma_c^* \bar{D}^0$ or $\Sigma_c^* \bar{D}^{*0}$ molecular states
- can pentaquarks form outside of decays?
- try hadronic rescattering!

A Prediction



Summary

- multi-parton interactions can be added via veto algorithm with parton shower
 - string model gives a relatively good description of hadronization
 - naive decays just require phase space
 - including spin correlations and proper structure much more complicated
 - once we have a full event, we can make a number of interesting predictions
- ① connect perturbative and non-perturbative regimes
 - ② provide complete events with final state particles
 - ③ robustly perform high-dimension integrals